

## Part III

### Administrative Procedures for Statistical Audit

26 CFR 601.105: Examination of returns and claims for refund or abatement; determination of correct tax liability.

(Also: Part I, § 6001)

Rev. Proc. 2011-42

#### SECTION 1. PURPOSE

This revenue procedure provides taxpayers with guidance regarding the use and evaluation of statistical samples and sampling estimates.

#### SECTION 2. BACKGROUND

The use of statistical sampling is provided in several items of published guidance. See, for example, Rev. Proc. 2011-35, 2011-25 I.R.B. 890 (provides safe harbor methodologies to determine basis in stock acquired in transfer basis transactions); Rev. Proc. 2004-29, 2004-1 C.B. 918 (provides the statistical sampling methodology that a taxpayer may use in establishing the amount of substantiated meal and entertainment expenses that are excepted from the 50% deduction disallowance under section 274(n)(1)); Rev. Proc. 2007-35, 2007-1 C.B. 1349 (addresses when statistical sampling may be used for purposes of section 199 (income attributable to domestic production activities)); Rev. Proc. 2002-55, 2002-2 C.B. 435 (permits external auditors of qualified intermediaries to use statistical sampling); and Rev. Proc. 72-36, 1972-2 C.B. 771 (sets forth statistical sampling guidelines for determining the redemption rate of trading stamps). If statistical sampling is determined to be appropriate under prior published guidance or under this revenue procedure, a taxpayer may use only the statistical sampling procedures set forth in

this revenue procedure. Thus, any published guidance in effect prior to the effective date of this revenue procedure that permits statistical sampling is modified and amplified by this revenue procedure.

### SECTION 3. SCOPE

When permitted by the Service, taxpayers may use statistical sampling procedures to support items on their income tax returns. The Service will use the criteria set forth in Section 4 of this revenue procedure in determining whether to accept a statistical sampling estimate as adequate substantiation for a return position. Statistical samples that fail to meet these criteria will be rejected.

### SECTION 4. APPLICATION

.01 In general. When permitted by the Service, a taxpayer may use statistical sampling in establishing, with respect to its income tax liability, items on its return by

following the procedures provided in Appendix A (Sampling Plan Standards), Appendix B (Sampling Documentation Standards) and Appendix C (Technical Formulas) of this revenue procedure.

.02 Evaluation of a probability sample: Two-step method.

(1) Appropriateness. The appropriateness of using a probability sample, to either support or be primary evidence of a tax amount, is a facts and circumstances determination. Factors to be used in determining whether a probability sample is appropriate include, but are not limited to, the following: the time required to analyze large volumes of data; the cost of analyzing data; and the other books and records that may independently exist or have greater probative value. Probability samples generally will not be considered appropriate if evidence is readily available from another source that can be demonstrated to provide a more accurate answer, or if the use of sampling does not conform to applicable financial accounting standards (e.g., Generally Accepted Accounting Principles (GAAP)).

(2) Validity of the final estimate. Second, taxpayers must determine whether the final estimate represents a valid estimate. In general, a final estimate will be considered valid (without regard to audit adjustment(s)) provided that all of the following conditions are met.

(a) Documentation. Taxpayers must maintain all of the proper documentation to support the statistical application, sample unit findings, and all aspects of the sample plan. Proper supportive documentation generally includes all of the information contained in Appendix A and Appendix B of this revenue procedure.

(b) Known chance of selection. The estimate must be based on a probability (i.e., statistical) sample, in which each sampling unit in the population has a known (non-zero) chance of selection, using either a simple random sampling method or stratified random sampling method.

(a) Taxpayer uses the least advantageous 95% one-sided confidence limit. The estimate must be computed at the least advantageous 95% one-sided confidence limit. The “least advantageous” confidence limit is either the upper or lower limit that results in the least benefit to the taxpayer. If the relative precision for a sampling plan, as described in section 4.03(4) of this revenue procedure, does not exceed 10%, the point estimate may be used in place of the least advantageous 95% one-sided confidence limit. When the relative precision is less than 15% and greater than 10%, the estimate will be computed as an amount between the least advantageous 95% one-sided confidence limit and the point estimate determined as follows:

$$\text{Estimate} = \text{Point Estimate} - (\text{Relative Precision} - .10)/.05 * (\text{Point Estimate} - \text{Least Advantageous 95\% One-Sided Confidence Limit})$$

Although many methods exist to estimate population values from the sample data, only the following estimators will be considered for acceptance: (i) Variable estimators permitted include the Mean (also known as the direct projection method); (ii) Difference (using “paired variables”); (iii) (combined) Ratio (using a variable of interest and a “correlated” variable); and (iv) (combined) Regression (using a variable of interest and a “correlated” variable). The first variable used for the difference, ratio and regression estimators must be the variable used in the mean estimator. The second variable used

for the difference, ratio, and regression estimators must be a variable that can be paired with the first variable and should be related to the first variable.

To be accepted by the Service as a method to estimate population values from the sample data, taxpayers who choose to use methods (iii) or (iv) described above must first demonstrate that the statistical bias inherent in those methods is negligible. See section 4.03(3) of this revenue procedure. The formulas for these estimators are provided in Appendix C and assume sampling without replacement. Attribute estimators permitted include (combined) proportion or total count.

#### .03 Variable Sampling Plans.

(1) Use estimate with smallest overall standard of error. Of all the final estimates determined as qualifying, the estimate with the smallest overall standard of error, as an absolute value, will generally be used (i.e., the size of the estimate is irrelevant in the determination of the value to be reported). Situations may exist when only a single estimator may be appropriate for the plan objective. In those specialized situations, the relevant estimator may be evaluated without consideration of other methods.

(2) Calculation of confidence limits. Confidence limits are calculated by adding and subtracting the precision of the estimate from the point estimate when precision is determined by multiplying the standard error by (i) the 95% one-sided confidence coefficient based on the Student's t-distribution with the appropriate degrees of freedom, or (ii) 1.645 (i.e., the normal distribution), assuming the sample size is at least 100 in each non-100% stratum.

(3) Demonstration of statistical bias. For either the (combined) Ratio or Regression methods (as described in section 4.02(2)(c)), in order to demonstrate that little statistical bias exists, the following applies after excluding all strata tested on a 100% basis (i.e., the entire population of a stratum is selected for evaluation): (i) the total sample size of all strata must be at least 100 units; (ii) each stratum for which a population estimate is made should contain at least 30 sample units; (iii) the coefficient of variation of the paired variable must be 15% or less; (iv) the coefficient of variation of the primary variable of interest, represented by either the corrected value or the difference between the reported and corrected values in common accounting situations, must be 15% or less; and (v) for only the (combined) Ratio method, the reported values of the units must be of the same sign. Definitional formulas for the paired variable (described in (iii) above), and the corrected value and the difference between the reported and corrected values (each described in (iv) above) are provided in Appendix C (Technical Formulas).

(1) Calculating the relative precision for each estimator. The relative precision for each estimator is commonly calculated by dividing the precision at the 95% one-sided confidence limit (sometimes referred to as sampling error) of the estimate by the estimator. When an estimate may be calculated using either a corrected value or difference perspective, as in the case of Ratio and Regression methods, or solely a corrected value perspective as in the case of a Mean method, the test will be applied on the basis of a difference perspective. In these cases the

numerator of the calculation is the sampling error of the adjustment and the denominator the point estimate of the adjustment.

(5) Specialized situations. For specialized situations, the 10% test that applies to the particular sampling objective must be appropriate for the plan, and adjusted accordingly to reflect an acceptable level of precision. Additional modifications may be necessary for other unique types of sampling plans.

(3) Exclusion of certain items for purposes of the 10% relative precision test. For the purpose of the 10% relative precision test, any stratum, when the sampling units or the process of evaluating the sampling units are different from those in other strata, must be excluded in calculating the relative precision.

#### .04 Attribute Sampling Plans

(4) Determining confidence limits. When using simple random samples, the confidence limits are determined using the Hypergeometric, Poisson, or Binomial distribution. If the proportion being estimated is between 30% and 70%, then the normal distribution approximation may be used in lieu of one of the above distributions. For stratified random samples, when at least two strata are sampled (i.e., not 100% samples), the confidence limits must be determined using the normal distribution approximation. If stratified random samples are not used, then confidence limits will be determined using the Hypergeometric, Poisson, or Binomial distribution.

(5) Calculating the relative precision. For the normal distribution approximation, the precision is calculated by multiplying the standard error by (i) the 95% one-sided confidence coefficient based on the Student's t-distribution with the

appropriate degrees of freedom, or (ii) 1.645 (i.e., the normal distribution), assuming the sample size is at least 100 in each non-100% stratum.

(4) Point estimate. One of the following two tests must be achieved for the use of the point estimate from an attribute sampling plan. A relative precision of 10% or less must be achieved on the point estimate (i.e., the estimated proportion,  $p$ ) and on its complement (i.e.,  $1 - p$ ). A simple random sample size of at least 300 must be used to determine the point estimate, when the sample size of 300 excludes dummy and null sampling units.

.05 Limitations.

(3) The Service's allowance of a taxpayer's estimate does not correspondingly require acceptance by the Service of the taxpayer's use of an estimate for the determination of associated adjustments, allocation, or subdivision of the findings for other purposes.

(4) This revenue procedure only addresses the statistical requirements that must be met for a probability sample to meet preliminary acceptance. It is not intended to further require acceptance of individual sample unit determinations. Valuation or attribute determinations remain subject to independent verification along with other non-statistical issues such as missing sampling items. Likewise, the statistical procedures followed may be examined and adjusted if the procedures are found to be in error. Any fatal error in statistical methodology that renders the probability sample invalid will preclude the use of any statistical estimate based on the sample and will only allow for consideration of the sample findings on an actual basis.



When a probability sample is determined to be not appropriate and is raised as an issue, the examining agent may pursue a more accurate determination or allow the findings of units examined on an actual basis. The computational validity of the estimator should still be considered and addressed along with other alternative issues in un-agreed cases.

(5) This revenue procedure does not preclude the Service from raising or pursuing any income, employment, or other tax issues identified in the review of a statistical sample.

(5) It is recognized that existing industry practices and specific taxpayers may be using techniques that are not covered by this revenue procedure. If a taxpayer employed a probability sample or method not covered by this revenue procedure, then the estimate may be referred to a Statistical Sampling Coordinator for resolution or issue development.

(6) This revenue procedure does not relieve taxpayers of their responsibility to maintain any documentation required by section 6001, other sections, or subsections that have specific documentation requirements for the entire population. Issues regarding documentation or support may be raised as appropriate.

(7) This revenue procedure does not supersede any specific rules for substantiation, such as those under section 274(d).

## SECTION 5. EFFECT ON OTHER DOCUMENTS

This revenue procedure modifies and amplifies Rev. Proc. 2007-35, Rev. Proc. 2004-29, Rev. Proc. 2002-55, and Rev. Proc. 72-36.

## SECTION 6. PAPERWORK REDUCTION ACT

The collection of information contained in this revenue procedure has been reviewed and approved by the Office of Management and Budget (OMB) in accordance with the Paperwork Reduction Act (44 U.S.C. 3507) under control number 1545-0123.

An agency may not conduct or sponsor, and a person is not required to respond to, a collection of information unless the collection of information displays a valid OMB control number.

The collection of information in this revenue procedure is in Appendix B. This information is required to ensure compliance with the statistical sampling methodology contained in this revenue procedure. The information will be used to evaluate compliance with the procedures described in this revenue procedure. The collection of information is mandatory. The likely recordkeepers are businesses or other for-profit institutions.

The estimated total annual recordkeeping burden is 2400 hours. The estimated annual burden per recordkeeper is 8 hours. The estimated number of recordkeepers is 300.

Books or records relating to a collection of information must be retained as long as their contents may be material in the administration of any internal revenue law. Generally tax returns and tax return information are confidential, as required by section 6103.



## **APPENDIX A**

### **Sampling Plan Standards**

Taxpayers are required to have a written sampling plan prior to the execution of a sample. The plan must include the following items:

- (1) The objective of the plan including a description of what value is being estimated and for which tax year(s) the estimate is applicable;
- (2) Population definition and reconciliation of the population to the tax return;
- (3) Definition of the sampling frame;
- (4) Definition of the sampling unit;
- (5) Source of the random numbers, the starting point or seed, and the method used in selecting them;
- (6) Sample size, along with supporting factors in the determination;
- (7) Method used to associate random numbers to the frame;
- (8) Steps to be taken to insure that the serialization of the frame is carried out independent of the drawing of random numbers;
- (9) Steps to be taken in evaluating the sampling unit; and
- (10) The appraisal method(s) to be used in appraising the sample.

**APPENDIX B****Sampling Documentation Standards****Sample Execution Documentation**

Taxpayers must retain adequate documentation to support the statistical application, sample unit findings, and all aspects of the sample plan and execution. The execution of the sample must be documented and include information for each of the following:

- (1) The seed or starting point of the random numbers;
- (2) The pairing of random numbers to the frame along with supporting information to retrace the process;
- (3) List of the sampling units selected and the results of the evaluation of each unit;
- (4) Supporting documentation such as notes, invoices, purchase orders, project descriptions, etc., which support the conclusion reached about each sample item;
- (5) The calculation of the projected estimate(s) to the population, including the computation of the standard error of the estimate(s);
- (6) A statement as to any slips or blemishes in the execution of the sampling procedure and any pertinent decision rules; and
- (7) Computation of all associated adjustments. (An example of an associated adjustment would be the amount of depreciation allowable based on a probability determination of an amount capitalized).

**APPENDIX C****Technical Formulas****UNSTRATIFIED (SIMPLE RANDOM SAMPLE)  
MEAN ESTIMATOR****STRATIFIED  
MEAN ESTIMATOR***Sample Mean of Audited Amounts*

$$\bar{x} = \frac{\sum x_j}{n}$$

*Estimate of Total Audited Amount*

$$\hat{X}_M = N \bar{x}$$

$$\hat{X}_{Ms} = \sum (N_i \bar{x}_i)$$

*Estimated Standard Deviation of the Audited Amount*

$$S_x = \sqrt{\frac{[\sum (x_j^2)] - n(\bar{x}^2)}{n-1}}$$

*Estimated Standard Error of the Total Audited Amount*

$$\hat{\sigma}(\hat{X}_M) = \frac{NS_x \sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{X}_{Ms}) = \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{xi}^2}{n_i} \right]}$$

*Achieved Precision of the Total Audited Amount*

$$A'_M = \frac{NU_R S_x \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{Ms} = U_R \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{xi}^2}{n_i} \right]}$$

**UNSTRATIFIED (SIMPLE RANDOM SAMPLE)  
DIFFERENCE ESTIMATOR**

**STRATIFIED  
DIFFERENCE ESTIMATOR**

*Estimate of Total Difference*

$$\hat{D} = N \bar{d}$$

$$\hat{D}_s = \sum(N_i \bar{d}_i)$$

*Estimate of Total Audited Amount*

$$\hat{X}_D = Y + \hat{D}$$

$$\hat{X}_{D_s} = Y + \hat{D}_s$$

*Estimated Standard Deviation of the Difference Amount*

$$S_D = \sqrt{\frac{[\sum (d_j^2)] - n (\bar{d}^2)}{n-1}}$$

*Estimated Standard Error of the Difference Amount*

$$\hat{\sigma}(\hat{D}) = \frac{NS_D \sqrt{1 - n/N}}{\sqrt{n}}$$

$$\hat{\sigma}(\hat{D}_s) = \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{D_i}^2}{n_i} \right]}$$

*Achieved Precision of the Difference Amount*

$$A'_D = \frac{NU_R S_D \sqrt{1 - n/N}}{\sqrt{n}}$$

$$A'_{D_s} = U_R \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{D_i}^2}{n_i} \right]}$$



**UNSTRATIFIED (SIMPLE RANDOM SAMPLE)  
RATIO ESTIMATOR**

**STRATIFIED  
COMBINED RATIO ESTIMATOR**

*Estimated Ratio of Audited Amount to Recorded Amount*

$$R = \frac{\sum x_j}{\sum y_j} = 1 + \frac{\sum d_j}{\sum y_j} \qquad \hat{R}_C = \frac{\sum (N_i \bar{x}_i)}{\sum (N_i \bar{y}_i)} = 1 + \frac{\sum (N_i \bar{d}_i)}{\sum (N_i \bar{y}_i)}$$

*Estimate of Total Audited Amount*

$$\hat{X}_R = Y\hat{R}$$

$$\hat{X}_{RC} = Y\hat{R}_C$$

*Estimated Standard Deviation of the Ratio*

$$S_R = \sqrt{\frac{\sum (x_j^2) + \hat{R}^2 \sum (y_j^2) - 2\hat{R} \sum (x_j y_j)}{n-1}}$$

*Estimated Standard Deviation of the Ratio in  $i^{\text{th}}$  Stratum*

$$S_{RC_i} = \sqrt{\frac{[(\sum x_{ij}^2 - (\sum x_{ij})^2 / n_i)] + [\hat{R}_C^2 (\sum y_{ij}^2 - (\sum y_{ij})^2 / n_i)] - [2\hat{R}_C (\sum x_{ij} y_{ij} - n_i \bar{x}_i \bar{y}_i)]}{n_i - 1}}$$

*Estimated Standard Error of the Ratio Amounts*

$$\hat{\sigma}(\hat{X}_R) = \frac{NS_R \sqrt{1 - n/N}}{\sqrt{n}} \qquad \hat{\sigma}(\hat{X}_{RC}) = \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{RC_i}^2}{n_i} \right]}$$

*Achieved Precision of the Ratio Amounts*

$$A'_R = \frac{NU_R S_R \sqrt{1 - n/N}}{\sqrt{n}} \qquad A'_{RC} = U_R \sqrt{\sum \left[ N_i (N_i - n_i) \frac{S_{RC_i}^2}{n_i} \right]}$$

**UNSTRATIFIED (SIMPLE RANDOM SAMPLE)  
REGRESSION ESTIMATOR**

**STRATIFIED  
COMBINED REGRESSION ESTIMATOR**

*Estimated Regression Coefficient*

$$b = \frac{[\sum(x_j y_j)] - n\bar{x}\bar{y}}{[\sum(y_j^2)] - n(\bar{y}^2)} = 1 + \frac{[\sum(d_j y_j)] - n\bar{d}\bar{y}}{[\sum(y_j^2)] - n(\bar{y}^2)} \quad b_c = \frac{\sum N_i(N_i - n_i)S_{XY_i}/n_i}{\sum N_i(N_i - n_i)S_{Y_i}^2/n_i} = 1 + \frac{\sum N_i(N_i - n_i)S_{DY_i}/n_i}{\sum N_i(N_i - n_i)S_{Y_i}^2/n_i}$$

*Estimate of Total Audited Amount*

$$\hat{X}_G = N\bar{x} + b(Y - N\bar{y}) \quad \hat{X}_{Gc} = \sum(N_i \bar{x}_i) + b_c[Y - \sum(N_i \bar{y}_i)]$$

*Estimated Standard Deviation of the Regression Amounts*

$$S_G = \sqrt{\frac{1}{n-2} \left[ [\sum(x_j^2)] - n(\bar{x}^2) - \frac{(\sum(x_j y_j) - n\bar{x}\bar{y})^2}{\sum(y_j^2) - n(\bar{y}^2)} \right]}$$

*Estimated Covariance between the Audited and Recorded Amounts in  $i^{\text{th}}$  Stratum*

$$S_{xy_i} = \frac{[\sum(x_{ij} y_{ij})] - n_i \bar{x}_i \bar{y}_i}{n_i - 1}$$

*Estimated Standard Deviation between the Audited and Recorded Amounts in  $i^{\text{th}}$  Stratum*

$$S_{Gc_i} = \sqrt{S_{X_i}^2 - 2b_c S_{XY_i} + b_c^2 S_{Y_i}^2}$$

*Estimated Standard Error of the Audited and Recorded Amounts*

$$\hat{\sigma}(\hat{X}_G) = \frac{NS_G \sqrt{1 - n/N}}{\sqrt{n}} \quad \hat{\sigma}(\hat{X}_{Gc}) = \sqrt{\sum \left[ N_i(N_i - n_i) \frac{S_{Gc_i}^2}{n_i} \right]}$$

*Achieved Precision of the Audited and Recorded Amounts*

$$A'_G = \frac{NU_R S_G \sqrt{1 - n/N}}{\sqrt{n}} \quad A'_{Gc} = U_R \sqrt{\sum \left[ N_i(N_i - n_i) \frac{S_{Gc_i}^2}{n_i} \right]}$$

### Definition of Symbols

TERM	DEFINITION
n	Sample Size
N	Population Size
x	The value of the sampling unit that is being used as the primary variable of interest. In audit sampling, this would be the audited (or revised) value of the transaction.
y	The value of the sampling unit that is being used as the “paired” variable that is related to the variable of interest. In audit sampling, this would be the reported (or original) value of the transaction.
d	The value of the sampling unit that is the difference between “paired” variable (y) and the variable of interest (x). That is, $d = x - y$ . In audit sampling, this would be the difference (or the change) of each transaction’s value.
X	The total value of the primary variable of interest. In audit sampling, this would be the estimated total audited value of the population. Typically, this value is not known for the entire population and is estimated based on the statistical sample selected.
Y	The total value of the variable that is paired with variable of interest. In audit sampling, this would be the total reported value of the population. Typically, this value is known for the entire population and may be estimated based on the statistical sample selected.
D	The total value of the difference between the “paired” variable and the variable of interest. In audit sampling, this would be the estimated total difference of the population. Typically, this value is not known for the entire population and is estimated based on the statistical sample selected.
U <sub>R</sub>	The confidence coefficient which is based on either the Student’s t-distribution or the normal distribution. For example, a 95% one-sided confidence coefficient based on the normal distribution is 1.645. This term is often referred to as the t-value and the z-value.